Expected Utility v.s Utility of Expected Wealth

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1 Examples

First, you should realize that these two concepts are all related to Utility and Expectations.

I believe one example can help you understand these two concepts well. Suppose that one agent with initial wealth \$2 is facing a gamble, he is expected to lose \$1 with probability $\frac{1}{4}$, gain \$2 with probability $\frac{1}{4}$, and nothing happens with probability $\frac{1}{2}$.

His utility is given by

$$U(w) = \sqrt{w}$$

Then what's his expected utility and what's his utility of expected wealth?

2 Utility of Expected Wealth

To find utility of expected wealth, we first need to find what's the expected wealth? It turns out to be the expectation of final wealths. Then what's his final wealth? Let's denote his final wealth by W. Then according to the example, his final wealth is

$$W = \begin{cases} (2-1) & w.p\ \frac{1}{4} \\ (2+2) & w.p\ \frac{1}{4} \\ 2 & w.p\ \frac{1}{2} \end{cases}$$

Thus the final wealth is

$$W = \begin{cases} 1 & w.p\ \frac{1}{4} \\ 4 & w.p\ \frac{1}{4} \\ 2 & w.p\ \frac{1}{2} \end{cases}$$

Then we see that W is a random variable, and the expected wealth is simply the expectation of W. Remember what we did in the last recitation, the expectation of W is

$$\begin{aligned} \mathbb{E}(W) &= \frac{1}{4}(1) + \frac{1}{4}(4) + \frac{1}{2}(2) \\ &= \frac{1}{4} + 1 + 1 = \frac{9}{4} \end{aligned}$$

Thus we found that expected wealth is simply $\frac{9}{4}$

Then what's utility of expected wealth? Since we know that his utility is $\sqrt{w}$, then his **utility of expected wealth** is simply

$$U(\mathbb{E}(W)) = \sqrt{\frac{9}{4}} \approx 1.5$$

3 Expected Utility

This also relates to his final wealth. Remember, his final wealth is

$$W = \begin{cases} 1 & w.p \ \frac{1}{4} \\ 4 & w.p \ \frac{1}{4} \\ 2 & w.p \ \frac{1}{2} \end{cases}$$

Then the **expected utility** (EU) is simply the expectation of utilities

$$\begin{aligned} EU &= \mathbb{E}\left(U\left(W\right)\right) \\ &= \left(\frac{1}{4}\right)\sqrt{1} + \frac{1}{4}\sqrt{4} + \frac{1}{2}\sqrt{2} \\ &= \frac{1}{4} + \frac{1}{2} + \frac{\sqrt{2}}{2} \\ &= \frac{3+2\sqrt{2}}{4} \approx 1.457 \end{aligned}$$

4 Comparison

So we find that

- **utility of expected wealth** is $U\left(\mathbb{E}\left(W\right)\right) = \sqrt{\frac{9}{4}} \approx 1.5$
- **expected utility** is $EU = \mathbb{E}\left(U\left(W\right)\right) \approx 1.457$

We found that expected utility is LESS than utility of expected wealth.
But Why?
Notice that since agent is risk-averse, thus by **Jensen's Inequality,** we have

$$U\left(\mathbb{E}\left(W\right)\right) \geq \mathbb{E}\left(U\left(W\right)\right)$$

Thus the utility of expected wealth $\mathbb{E}\left(U\left(W\right)\right)$ is greater than expected utility $\mathbb{E}\left(U\left(W\right)\right)$ if agent is risk averse

(I hope you did not forget the Jensen's inequality I covered in the recitaion (remember the graph I draw on the board), this result is simply an application).

On the other hand, if agent is risk lover, then

$$U\left(\mathbb{E}\left(W\right)\right) \leq \mathbb{E}\left(U\left(W\right)\right)$$